CSIR NET Life Science Unit 5

Lotka-Volterra Model of Interspecific Competition

Introduction

The Lotka-Volterra model was independently derived by two scientists, Alfred Lotka and Vito Volterra. In 1920, Alfred Lotka studied the predator-prey interactions and observed that system of two biological species could oscillate permanently. He published his finding in "Analytical note on certain rhythmic relations in organic systems". In 1926, Vito Volterra an Italian mathematician and physicist published "Fluctuation in the abundance of a species considered mathematically" to study the evolution of predator and prey fish populations in the Adriatic Sea. Lotka-Volterra model is based on two different mechanisms one is that of predator and prey, while the other one is of interspecific competition.

Interspecific competition

Lotka-Volterra model can theoretically give us the outcome of interspecific competition between two species by the initial population size N, the carrying capacity K and the competition coefficient.
The equation of Lotka-Volterra which is for the competition is based on a logistical curve. So, the equation for the logistical curve for the species is as follows, For species 1-

$$\frac{dN_1}{dt} = r_1N_1\left(\frac{K_1 - N_1}{K_1}\right) \quad \text{eq (1)}$$

For species 2-

$$\frac{dN_2}{dt} = r_2N_2\left(\frac{K_2 - N_2}{K_2}\right) \quad \text{eq (2)}$$

where, where, N = population size of the speciest= time r= intrinsic carrying capacity for the increase of the speciesK= carrying capacity for the species The two species interact with each other and compete for resources which will affect each other's population growth. The critical resources in the environment will not inevitably be essential for both the species, for example if species 1 uses more resources than species 2, then the environment will hold K1 individual of species when they utilize the resource. But if species 2 utilizes a very less amount for its survival. To define this competitive situation a conversion factor is used, where an assumption is made that under all conditions of density there is a conversion factor between the competitors also known as the competition coefficient.

$\alpha N2$ = equivalent number of species 1 individuals
Similarly, for species 2, a conversion factor will be used to make it equivalent to species 1
$\beta N1$= equivalent number of species 2 individuals so, the equation now will be

$$\frac{dN_1}{dt} = r_1N_1\left(\frac{K_1 - N_1 - \alpha N_2}{K_1}\right)$$ for species 1 eq (3)

$$\frac{dN_2}{dt} = r_2N_2\left(\frac{K_2 - N_2 - \beta N_1}{K_2}\right)$$ for species 2 eq (4)

When both the species compete together there are three possible outcomes When both the species compete together there are three possible outcomes • Both the species will co-exist• Species 1 becomes extinct• Species 2 becomes extinctSolving equations 3 and 4 at equilibrium that is

$$\frac{dN_1}{dt} = 0 = \frac{dN_2}{dt}$$

Four possible outcomes will come for the competing species as given below in the figure. When there is no equilibrium that is the diagonals don't cross each other as in case 1 and case 2 this leads to extinction. Whereas the crossing gives us two points: one is the stable equilibrium where both the species exist and an unstable point where even a slight change or disturbance may lead to the extinction of one of the species. While in case 4 if the population is slightly downward, they will reach a zone where N1 can increase and N2 will decrease which will result in species 1 coming to equilibrium by itself at K1.

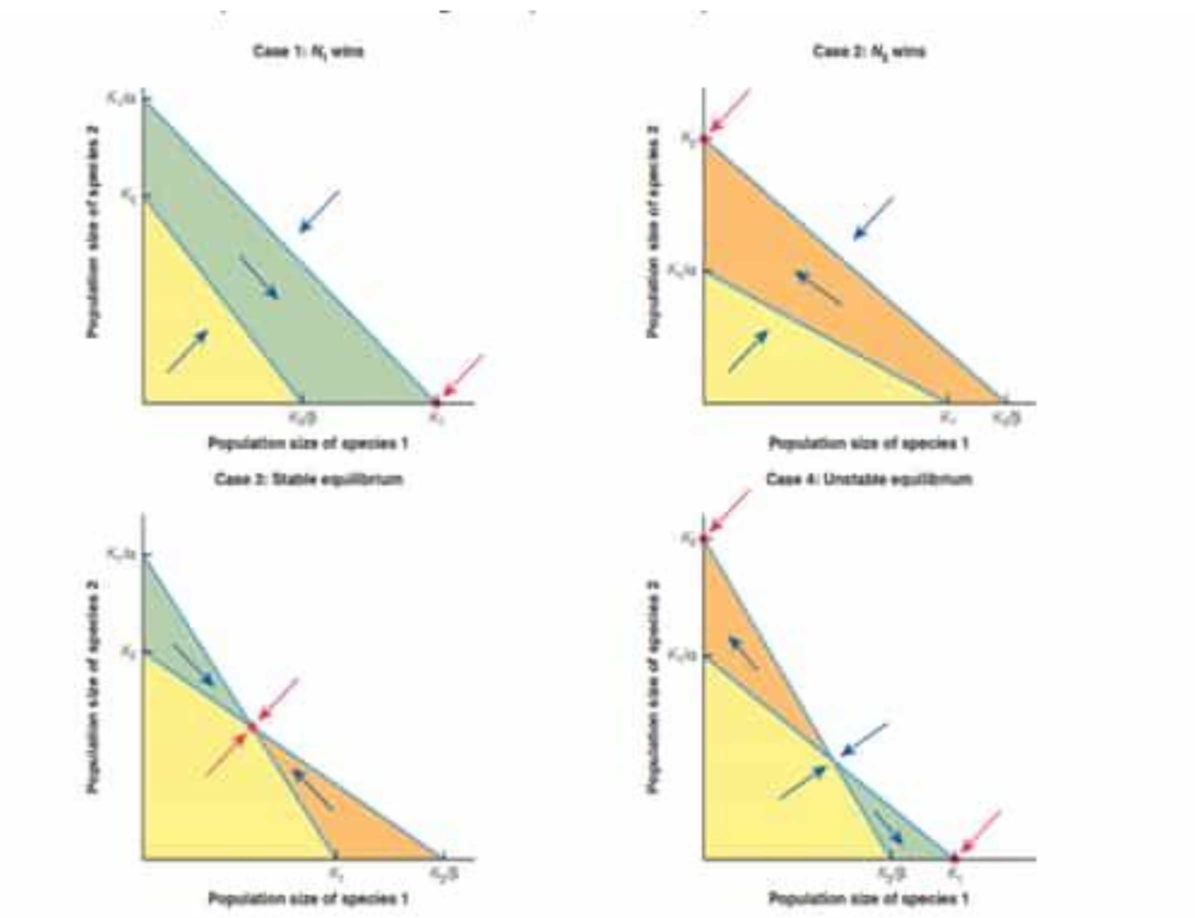

Figure 1. Possible outcomes of two competing species, where the blue arrow indicates the growth of the population and the red dot and red arrow show the equilibrium points. In the yellow zone both species increase, the green zone is only for species 1 and orange for species 2 whereas the white zone is where both the species decrease.